WeSchool Knowledge Builder - The National Journal 2015
Volume -3 | 2015 | ISBN 978-1-63102463-4
weschool.editor@rtmonline.in |http://weschool.rtmonline.in | http://www.rtmonline.in

FLUID DYNAMICS AND ITS PRINCIPLES: A STUDY OF MATHEMATICAL APPLICATIONS

by

Ismail Hussain | Research Scholar | Maharishi University Of Information Technology, Lucknow, Uttar Pradesh

&

Dr. V. K. Rathaur (Ph.D in Mathematics) | Research Supervisior | Maharishi University of Information Technology, Lucknow, Uttar Pradesh

ABSTRACT

Fluid- dynamics is an "ancient science" that is incredibly alive today. The modern technologies require a deeper understanding of the behavior of real fluids; on the other hand new discoveries often pose new challenging mathematical problems. In this framework a special role is played by incompressible viscous flows. The study of these flows has been attached with a wide range of mathematical techniques and, to day, this is a stimulating part of both pure an applied mathematics. Fluid is a material that is infinitely deformable or malleable. A fluid may resist moving from one shape to another but resists the same amount in all directions and in all shapes. The basic characteristic of the fluid is that it can flow.

KEYWORDS: Fluid- dynamics, technologies, mathematical, problems, techniques, material, directions, shapes, characteristic, flow, Principles, etc.

INTRODUCTION

Fluid motions are generally classified into three groups: Laminar flows, Laminar-Turbulent transition flows and Turbulent flows. Laminar flow is the stream-lined motion of the fluid, while the turbulent flow is random in space and time, while the laminar-turbulent transition concerns unstable flows. In order to indicate the path along which the fluid is flowing we can use the streamlines [1]. So, streamlines are those lines that the tangent at a certain point on it gives the direction of the fluid velocity at that point.

It already by the time of the Roman Empire enough practical information had been accumulated to permit quite sophisticated applications of fluid dynamics [4-5]. The more modern understanding of fluid motion began several centuries ago with the work of L. Euler and the Bernoullis (father and son), and the equation we know as Bernoulli's equation (although this equation was probably deduced by someone other than a Bernoulli). The equations derived and study in lectures which was introduced by Navier in the 1820s, and the complete system of equations representing essentially all fluid motions were given by Stokes in the 1840s. These are now known as the Navier–Stokes equations, and they are of crucial importance in fluid dynamics [2]. For most of the 19th and 20th Centuries there were two approaches to the study of fluid motion: theoretical and experimental. Many contributions to our understanding of fluid behavior were made through the years by both of these methods. But today, because of the power of modern digital computers, there is yet a third way to study fluid dynamics: computational fluid dynamics, or CFD for short. In modern industrial practice CFD is used more for fluid flow analyses than either theory or experiment. Most of what can be done theoretically has already been done, and experiments are generally difficult and expensive. But it is also important to understand that in order to do CFD one must have a fundamental understanding of fluid flow itself, from both the theoretical, Theoretical/analytical studies of fluid dynamics generally require considerable simplifications of the equations of fluid motion mentioned [3].

REVIEW OF LITERATURE

A fluid is anything that flows, usually a liquid or a gas, the latter being distinguished by its great relative compressibility. Fluids are treated as continuous media, and their motion and state can be specified in terms of the velocity u, pressure p, density ρ, etc. evaluated at every point in space x and time t. To define the density at a point, for example, suppose the point to be surrounded by a very small element (small compared with length scales of interest in experiments) which nevertheless contains a very large number of molecules [6]. The density is then the total mass of all the molecules in the element divided by the volume of the element. Considering the velocity, pressure, etc. as functions of time and position in space is consistent with measurement techniques using fixed instruments in moving fluids. It is called the Eulerian specification. However, Newton's laws of motion (see below) are expressed in terms of individual particles, or fluid elements, which move about. Specifying a fluid motion in terms of the position X (t) of an individual particle (identified by its initial position, say) is called the Lagrangian specification [7]. The two are linked by the fact that the velocity of such an element is equal to the velocity of the fluid evaluated at the position occupied by the element:

$$\frac{d\mathbf{X}}{dt} = \mathbf{u}[\mathbf{X}(t), t] \ .$$

The path followed by a fluid element is called a particle path, while a curve which, at any instant, is everywhere parallel to the local fluid velocity vector is called a streamline. Particle paths are coincident with streamlines in steady flows, for which the velocity u at any fixed point x does not vary with time t.

Newton's Laws refer to the acceleration of a particle. A fluid element may have acceleration both because the velocity at its location in space is changing (local acceleration) and because it is moving to a location where the velocity is different (convective acceleration) [8]. The latter exists even in a steady flow. How to evaluate the rate of change of a quantity at a moving fluid element, in the Eulerian specification? Consider a scalar such as density ρ (x, t). Let the particle be at position x at time t, and move to x + $\delta\mathbf{x}$ $t + \delta t$,at time where (in the limit of small dt)

$$\delta\mathbf{x} = \mathbf{u}(\mathbf{x}, t)\,\delta t \ .$$

Then the rate of change of ρ following the fluid, or material derivative, is

$$\frac{D\rho}{Dt} = \lim_{\delta t \to 0} \frac{\rho(\mathbf{x} + \delta\mathbf{x}, t + \delta t) - \rho(\mathbf{x}, t)}{\delta t}$$

$$= \frac{\partial\rho}{\partial x}\frac{\delta x}{\delta t} + \frac{\partial\rho}{\partial y}\frac{\delta y}{\delta t} + \frac{\partial\rho}{\partial z}\frac{\delta z}{\delta t} + \frac{\partial\rho}{\partial t}$$

(By the chain rule for partial differentiation)

$$= \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} \qquad (3a)$$

(Using (2))

$$= \frac{\partial \rho}{\partial t} + \mathbf{u}.\nabla\rho \qquad (3b)$$

In vector notation, where the vector ∇ ρ is the gradient of the scalar field ρ:

$$\nabla\rho = \left(\frac{\partial \rho}{\partial x}, \frac{\partial \rho}{\partial y}, \frac{\partial \rho}{\partial z}\right).$$

A similar exercise can be performed for each component of velocity, and we can write the x-component of acceleration as

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}, \qquad (4a)$$

Etc. combining all three components in vector shorthand we write

$$\frac{D\mathbf{u}}{Dt} = \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u}, \qquad (4b)$$

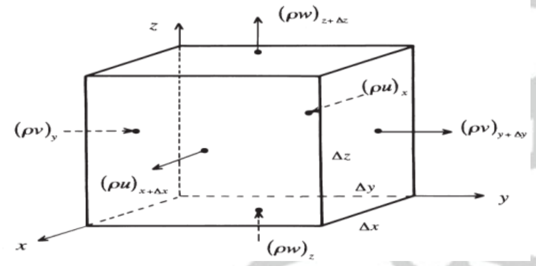

FIG. 1 – Mass flow into and out of a small rectangular region of space

But care is needed because the quantity ∇u is not defined in standard vector notation. Note that ∂u/∂t is the local acceleration, (u.∇) u the convective acceleration. Note too that the convective acceleration is nonlinear in u, which is the source of the great complexity of the mathematics and physics of fluid motion.

1. Conservation of mass:

This is a fundamental principle, stating that for any closed volume fixed in space, the rate of increase of mass within the volume is equal to the net rate at which fluid enters across the surface of the volume [9]. When applied to the arbitrary small rectangular volume depicted in fig. 1, this principle gives:

$$\Delta x\Delta y\Delta z\frac{\partial \rho}{\partial t} = \Delta y\Delta z\left([\rho u]_x - [\rho u]_{x+\Delta x}\right) +$$

$$+\Delta z\Delta x\left([\rho v]_y - [\rho v]_{y+\Delta y}\right) +$$

$$+\Delta x\Delta y\left([\rho w]_z - [\rho w]_{z+\Delta z}\right).$$

Dividing by Δx Δy Δ z and taking the limit as the volume becomes very small we get

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x}(\rho u) - \frac{\partial}{\partial y}(\rho v) - \frac{\partial}{\partial z}(\rho w) \qquad (5a)$$

Or (in shorthand)

$$\frac{\partial \rho}{\partial t} = -\mathrm{div}(\rho\mathbf{u}) \qquad (5b)$$

2. The Navier-Stokes equations

❖ **Newton's Laws of Motion**

Newton's first two laws state that if a particle (or fluid element) has an acceleration then it must be experiencing a force (vector) equal to the product of the acceleration and the mass of the particle:

Force = mass × acceleration.

For any collection of particles this becomes net force = rate of change of momentum where the momentum of a particle is the product of its mass and its velocity. Newton's third law states

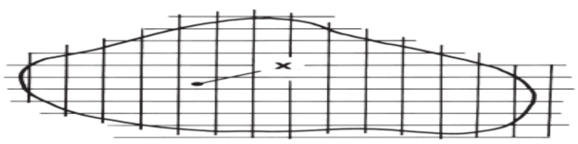

FIG. 2 – An arbitrary region of fluid divided up into small rectangular elements (depicted only in two dimensions).

that, if two elements A and B exert forces on each other, the force exerted by A on B is the negative of the force exerted by B on A. To apply these laws to a region of continuous fluid, the region must be thought of as split up into a large number

Volume -3 | 2015 | ISBN 978-1-63102463-4
weschool.editor@rtmonline.in |http://weschool.rtmonline.in | http://www.rtmonline.in

of small fluid elements (fig. 2), one of which, at point x and time t, has volume ΔV , say. Then the mass of the element is ρ (x,t) ΔV , and its acceleration is Du/Dt evaluated at (x,t).

3. Body force and stress:

The force on an element consists in general of two parts, a body force such as gravity exerted on the element independently of its neighbours, and surface forces exerted on the element by all the other elements (or boundaries) with which it is in contact. The gravitational body force on the element ΔV is g ρ (x, t) ΔV, where g is the gravitational acceleration. The surface force acting on a small planar surface, part of the surface of the element of interest, can be shown to be proportional to the area of the surface, ΔA say, and simply related to its orientation, as represented by the perpendicular (normal) unit vector n (fig. 3). The force per unit area, or stress, is then given by

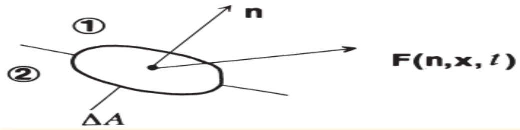

FIG. 3 – Surface force on an arbitrary small surface element embedded in the fluid, with area ΔA and normal n. F is the force exerted by the fluid on side 1, on the fluid on side 2.

$$F_x = \sigma_{xx} n_x + \sigma_{xy} n_y + \sigma_{xz} n_z$$
$$F_y = \sigma_{yx} n_x + \sigma_{yy} n_y + \sigma_{yz} n_z \qquad (9a)$$
$$F_z = \sigma_{zx} n_x + \sigma_{zy} n_y + \sigma_{zz} n_z$$

Or, in shorthand,

$$\mathbf{F} = \underset{\approx}{\sigma}\, \mathbf{n} \qquad (9b)$$

Where $\underset{\approx}{\sigma}$ is a matrix quantity, or tensor, depending on x and t but not n or ΔA. $\underset{\approx}{\sigma}$ is called the stress tensor, and can be shown to be symmetric (i.e. $\sigma_{yx} = \sigma_{xy}$ etc.) so it has just 6 independent components. It is an experimental observation that the stress in a fluid at rest has a magnitude independent of n and is always parallel to n and negative, i.e. compressive. This means that

$$\sigma_{xy} = \sigma_{yz} = \sigma_{zx} = 0, \; \sigma_{xx} = \sigma_{yy} =$$

$\sigma_{zz} = -p,$ say, where p is the positive pressure (hydrostatic pressure); alternatively,

$$\underset{\approx}{\sigma} = -p\, \underset{\approx}{\mathbf{I}} \qquad (10)$$

Where $\underset{\approx}{\mathbf{I}}$ is the identity matrix? The relation between stress and deformation rate in a moving fluid, the motion of a general fluid element can be thought of as being broken up into three parts: translation as a rigid body, rotation as a rigid body, and deformation [10].

Quantitatively, the translation is represented by the velocity field u; the rigid rotation is represented by the curl of the velocity field, or vortices.

CONCLUSION

The aim of this paper is to furnish some results in very different areas that are linked by the common scope of giving new insight in the field of fluid dynamics. Since the arguments treated are various, an extensive bibliography has been added. For the sake of completeness, there is an introductory chapter and each subsequent new topic is illustrated with the will of a self-contained exposition.

In this framework a special role is played by incompressible viscous flows. The study of these flows has been attached with a wide range of mathematical techniques and, today, this is a stimulating part of both pure an applied mathematics.

REFERENCES

1. Babuˇska, I., Oden, J. T., and Baumann, C. E. (1999). Math. Appl., 37, 103–122.
2. Dolejˇs´i, V. (1998). Anisotropic mesh adaptation for finite volume and finite element methods on triangular meshes. Comput. Visualization Sci., 1(3), 165–178.
3. Dolejˇs´i, V. (2001). Anisotropic mesh adaptation technique for viscous flow simulation. East–West J. Numer. Math., 9(1), 1–24.
4. Dolejˇs´i, V. (2004). On the discontinuous Galerkin method for the numerical solution of the Euler and Navier–Stokes equations. Int. J. Numer. Meth. Fluids.
5. Dolejˇs´i, V. and Feistauer, M. (2003). The Preprint Series of the School of Mathematics MATH-KNM-2003/2, Charles University, Prague.
6. Dolejˇs´i, V., Feistauer, M., Felcman, J., and Klikov´a, A. (2002). Appl. Math., 47(4), 301– 340.
7. Dolejˇs´i, V., Feistauer, M., and Schwab, C. (2002). A finite volume discontinuous Galerkin scheme for nonlinear convection–diffusion problems. Calcolo, 39, 1– 40.
8. Dolejˇs´i, V., Feistauer, M., and Sobot´ikov´a, V. (2005). Comput. Methods. Appl. Mech. Engrg., 194, 2709–2733.
9. Eymard, R., Gallou¨et, T., and Herbin, R. (2000). Finite Volume Methods, pp. 717–1020. NorthHolland-Elsevier, Amsterdam.
10. Feireisl, E., Novotn´y, A., and Petzeltov´a, H. (2001). J. Math. Fluid Mech., 3, 358–392.