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TRANSFORMATION TECHNIQUE TO SOLVE MULTI-OBJECTIVE LINEAR FRACTIONAL PROGRAMMING PROBLEM

by

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ABSTRACT

In mathematical optimization, linear-fractional programming (LFP) is a generalization of linear programming (LP). As the objective functions in linear programs are linear functions, the objective function in a linear-fractional program is a ratio of two linear functions. A linear program can be regarded as a special case of a linear-fractional program in which the denominator is the constant function one. Multi-objective linear fractional programming (MOLFP) technique is a very useful decision-making tool for modeling problems with more than one objective such as profit/cost, actual cost/standard cost, debt/equity, inventory/sales, etc. subject to the system constraints. MOLFP is applied to different real-life problems such as agricultural planning problem, production planning problem, inventory problem & other problems.

KEYWORDS: Mathematical, problems, optimization, Transformation Technique, Multi-Objective, Linear etc.

INTRODUCTION

Optimization is the science of selecting the best of many possible decisions in a complex real life situation. The ultimate target of all such decisions is to either maximize the desired benefit or to minimize the effort required, incurred in a certain course of action [1]. The systematic approach to decision making generally involves three closely interrelated stages. The first stage towards optimization is to express the desired benefits, the required efforts and collecting the other relevant data, as a function of certain variables that may be called "decision variables" [2-4]. The second stage continues the process with an analysis of the mathematical model and selection of appropriate numerical technique for finding the optimal solution. The third stage consists of finding an optimal solution, generally with the help of computer.

The existence of optimization problem can be traced to the middle of eighteenth century. The work of Newton, Lagrange and Cauchy in solving certain types of optimization problems arising in physics and geometry by using differential calculus methods and calculus of variations is pioneering. These optimization techniques are known as classical optimization techniques and can be generalized to handle cases in which the variables are required to be non-negative and constraints may be inequalities, but these generalizations are primarily of theoretical value and do not usually constitute computational procedures [5]. However, in some simple situations they can provide solutions, which are practically acceptable. The optimization problems that have been posed and solved in the recent years have tended to become more and more elaborate, not to say abstract. Perhaps the most outstanding example of the rapid development of the optimization techniques occurred with the dynamic programming by Bellman in 1957 and of the maximum principle by Pontryagin in 1958, and the techniques were designed to solve the problems of the optimal control of dynamic systems. The simply stated problem of maximizing or minimizing a given function of several variables attracted the attention of many mathematicians over the past sixty years or so for developing the solution techniques under mathematical programming.

REVIEW OF LITERATURE

Optimization covers a wide range of examples and applications. Some typical fields where the problems of optimization arise frequently are the industry, Economics, Commerce, Aerodynamics etc. Over the second half of the 20th century, optimization found widespread applications in the study of physical and chemical systems, production planning and scheduling systems, location and transportation problems, resource allocation in financial systems, and engineering design [6]. From the very beginning of the application of optimization to these problems, it was recognized that analysts of natural and technological systems are almost always confronted with uncertainty such as the prices of fuels, the availability of electricity, and the demand for chemicals. A key difficulty in optimization under uncertainty is in dealing with an uncertainty space that is huge and frequently leads to the very large-scale optimization models. Decision-making under uncertainty is often further complicated by the presence of integer decision variables to model logical and other discrete decisions in a multi-period or multi-stage setting.

The development of the concepts of linear and nonlinear optimization models presumes that all of the data for the optimization model are known with certainty. However, uncertainty and inexactness of data and outcomes pervade many aspects of most optimization problems [7]. As it turns out, when the uncertainty in the problem is of a particular (fairly general) form, it is relatively easy to incorporate the uncertainty into the optimization model.

1. Mathematical Programming:

A large number of real-life optimization problems that are usually not solvable by classical optimization methods are formulated as mathematical programming problems. There has been considerable advancement towards the development of the theory and algorithms for solving various types of mathematical programming problems [8].

The first mathematical programming problem was considered by economists (Von Neumann) in early 1930s, as the problem of optimum allocation of limited resources. Leontief in 1951 showed a practical solution method for linear type problems when demonstrated his input-output model of an economy. These economic solution procedures did not provide optimal solution, but only a feasible solution, providing the model's linear constraints. In 1941, Hitchcock formulated and solved the transportation type problem, which was also

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accomplished by Koopmans in 1949. In 1942, Kantorovich also formulated the transportation problem but did not solve it. In 1945, the economist G. J. Stigler formulated and solved the "minimum cost diet" problem. During World War II a group of researchers under the direction of Marshall K. Wood sought to solve allocation type problem for the United States Air Force team SCOOP (Scientific Computation of Optimum Programs). One of the members of this group, George B. Dantzig, formulated and devised a solution procedure in 1947 for Linear Programming Problems (LPP). This solution procedure, called the Simplex method, marked the beginning of the field of study called mathematical programming. During the 1950s other researchers such as David Gale, H.W. Kuhn and A.W. Tucker contributed to the theory of duality in LP. Others such as Charnes and Cooper contributed numerous LP applications illustrating the use of mathematical programming in managerial decision-making.

A general Mathematical Programming Problem can be stated as following:

$Max\ (\ or\ Min\ )\ \ Z = f\,(X\,)$ (1.1)

$Subject\ to\ \ g_i\,(X\,) \le or = or \ge b_i \quad \forall\ i = 1, 2,..., m,$ (1.2)

Where

X =Vector of unknown variables that are subject to the control of decision maker.

Z =Value of the objective function which measures the effectiveness of the decision choice.

$g_i\ (X\ )$ = The function representing the i^{th} constraint, i =1,..., m b_i = available i^{th} productive resource in limited supply, i =1,...,

2. Linear Programming Techniques:

The general approach to the modeling and solution to linear mathematical models, and more specifically those models that seek to optimize a linear measure of performance, under linear constraints is called as linear programming or more appropriately as linear optimization [9]. The general form of the (single objective) Linear Programming Problem (LPP) is given as to find the decision variables x_1, x_2... x_p, which maximize or minimize a linear function, subject to some linear constraints and the non-negativity restrictions on the decision variables. Mathematical model for a general linear programming problem is stated as follows:

Max (or Min) Z = ∑p =1

Subject to ∑p aij x j ≤or =or ≥ bi i =1, 2,..., m
j=1

x j ≥0 j = 1, 2,..., p

Where c_j , a_{ij} and b_i (called parameters of the LPP) are known constants for all i and j .

3. Nonlinear Programming Techniques:

The mathematical model that seeks to optimize a non-linear measure of performance is called non-linear program. Every real world optimization problem has always a non-linear form which becomes a linear programming problem after a slight modification. Nonlinear programming emerges as an increasingly important tool in economic studies and in operations research. Nonlinear programming problems arise in various disciplines as engineering, business administration, physical sciences and in mathematics or in any other area where decision must be taken in some complex situation that can be represented by a mathematical model:

Minimize $f\,(x)$, *Subject to* $g_i\,(x) \ge 0$, i =1, 2,..., m

Where all or some of the functions $f(x)$ and $g_i(x), i = 1,..., m$ are non-linear.

4. Integer Programming:

Any decision problem (with an objective to be maximized or minimized) in which the decision variables must assume non fractional or discrete values may be classified as an integer optimization problem. In general, an integer problem may be constrained or unconstrained and the functions representing the objective and constraints may be linear or nonlinear [10]. An integer problem is classified as linear if by relaxing the integer restriction on the variables, the resulting functions are strictly linear.
The general mathematical model of an integer-programming problem can be stated as:
Maximize (or Minimize)
$Z = f(X)$

Subject to $g_i(X)\{\leq or = or \geq\} b_i$,
$i = 1,2,....., m$.
$x_j \geq 0,$ $j = 1,2,....,n$
x_j is an integer for $j \in J \subseteq I = (1,2,...., n)$

Where X =(x_1... x_n) is n-component vector of decision variables

If $J = I$, that is, all the variables are restricted to be integers we have an all (or pure) integer programming problem (AIPP) Otherwise, if $J \subset I$, *i.e.*, not all the variables are restricted to be integers, we have a mixed integer-programming problem (MIPP). In most of the practical situations the values of the decision variables are required to be integers. Dantzig, Fulkerson and Johnson (1954), Markowitz and Manne (1957), Dantzig (1958, 1959), etc. discussed the integer solutions to some special purpose LPPs. Gomory (1956) suggested first of all the systematic method to obtain an optimal integer solution to an AIPP. Later, Gomory (1960, 1963) extended the method to deal with the more complicated case of MIPP, when only some of the variables are required to be integers. These algorithms are proved to converge to the optimal integer solution in a finite number of iterations making use of familiar dual simplex method. These are called the "cutting plane algorithms" because they mainly introduce the clever idea of constructing "secondary" constraints which, when added to the optimum (non-integer) solution, will effectively cut the solution space towards the required result.

CONCLUSION

In this study, we present a transformation method for solving linear fractional programming problem when the objective function is ratio function and the set of constraints is in the form of linear inequality. Our proposed method based upon transformation technique. Our new method can be applied to any linear fractional programming problem, since it is a special thing of the mathematical program. In line with this, we discussed FGP models for dealing with MLFPP to seek compromise optimal solution. The pro-posed approach is also extended to solve MO-MLFPP. Alternative FGP models for MO-MLFPP are also proposed in this study. Illustrative numerical examples are solved in order to show the effectiveness of the proposed FGP approach.

REFERENCES

1. Aggarwal, O.P. (1974a): On mixed integer quadratic problems, *Naval Research Logistics Quarterly*, **21**, pp. 289-297.
2. Aggarwal, O.P. (1974b). On integer solution to quadratic problems by a

branch and bound technique., *Trabajas de Estatistica Y De Investigation operation*, **25**, pp. 65-70.

3. Ahsan, M.J. and Khan, S.U. (1977). Optimum allocation in multivariate stratified random sampling using prior information. *J. Ind. Stat. Assoc.*, **15**, pp. 57-67.
4. Ahsan, M.J., Khan, S.U. and Arshad, M. (1983). Minimising a non-linear function arising in stratification through approximation by quadratic function. *J. Ind. Stat. Assoc. Oper. Res.*, **4**, pp. 9-16.
5. Alidaee, B. Kochenberger, G.A. and Ahmadin, A. (1994). 0-1 quadratic programming approach for optimum solution of two scheduling problems. *Internet J. Systems Sci*., **25**, **(2)**, pp. 401-408.
6. Amstader, B. (1971), “Reliability Mathematics,” New York: McGraw-Hill.
7. Anstreicher. K.M., and Brixius, N.W., (2001). A new bound for the quadratic assignment problem based on convex quadratic programming. *Math. Prog. Ser*. A, **89**, **(3)**, pp. 341-357.
8. Anstreicher, K.M., den Hertog, D. and Terlaky, T. (1994). A long-step barrier method for convex quadratic programming. *Algorithmica*, **20**, pp. 365-382.
9. Arabeyre, J.P., Farnley, J. and et al., (1969). The airline crew scheduling problem: A survey. *Transportation Science*, **3**, pp. 140-163.
10. Arshad, M., Khan, S.U. and Ahsan, M.J. (1981). A procedure for solving concave quadratic programs. *Aligarh Journal of Statistics*, **1**, **(2)**, pp. 106-112.